NAG Toolbox for MATLAB

f07hr

1 Purpose

f07hr computes the Cholesky factorization of a complex Hermitian positive-definite band matrix.

2 Syntax

```
[ab, info] = f07hr(uplo, kd, ab, 'n', n)
```

3 Description

f07hr forms the Cholesky factorization of a complex Hermitian positive-definite band matrix A either as $A = U^{H}U$ if **uplo** $=$ 'U' or $A = LL^{H}$ if **uplo** $=$ 'L', where U (or L) is an upper (or lower) triangular band matrix with the same number of superdiagonals (or subdiagonals) as A.

4 References

Demmel J W 1989 On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo $=$ 'U'

The upper triangular part of A is stored and A is factorized as $U^{H}U$, where U is upper triangular.

uplo $=$ 'L'

The lower triangular part of A is stored and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2: **kd – int32 scalar**

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab(ldab,$*$) – complex array**

The first dimension of the array **ab** must be at least **kd** $+ 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n Hermitian positive-definite band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** $=$ 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j,j)$ for $\max(1j - k_d) \leq i \leq j$;
if **uplo** $=$ 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j,j)$ for $j \leq i \leq \min(nj + k_d)$.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab

5.4 Output Parameters

1: **ab(ldab,*) – complex array**

The first dimension of the array **ab** must be at least $\mathbf{kd} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of A contains the Cholesky factor U or L as specified by **uplo**, using the same storage format as described above.

2: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **ab**, 5: **ldab**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

$\mathbf{info} > 0$

If $\mathbf{info} = i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. There is no function specifically designed to factorize a Hermitian band matrix which is not positive-definite; the matrix must be treated either as a nonsymmetric band matrix, by calling f07br or as a full Hermitian matrix, by calling f07mr.

7 Accuracy

If **uplo** $=$ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(k + 1)\epsilon|U^{\mathrm{H}}||U|,$$

$c(k + 1)$ is a modest linear function of $k + 1$, and ϵ is the ***machine precision***.

If **uplo** = 'L', a similar statement holds for the computed factor L. It follows that $|e_{ij}| \leq c(k+1)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $4n(k+1)^2$, assuming $n \gg k$.

A call to f07hr may be followed by calls to the functions:

f07hs to solve $AX = B$;

f07hu to estimate the condition number of A.

The real analogue of this function is f07hd.

9 Example

```
uplo = 'L';
kd = int32(1);
ab   =   [complex(9.39,   +0),   complex(1.69,   +0),   complex(2.65,   +0),
complex(2.17, +0);
      complex(1.08, +1.73), complex(-0.04, -0.29), complex(-0.33, -2.24),
complex(0, 0)];
[abOut, info] = f07hr(uplo, kd, ab)
```

```
abOut =
   3.0643             1.1167             1.6066             0.4289
   0.3524 + 0.5646i  -0.0358 - 0.2597i  -0.2054 - 1.3942i        0
info =
           0
```